DIOPHANTINE ANALYSIS.

85. Proposed by A. H. BELL, Hillsboro, Ill.

Given $x^2-85\frac{1}{4}y^2=5$. What is the value of x and y in whole numbers?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

$x^2-85\frac{1}{4}y^2=5$, $4x^2-341y^2=20$, $x^2=5+\frac{341}{4}y^2$.

Let $y=2z$. $\therefore$ $x^2=5+341z^2$.

Let $z=2$. Then $x=37$, $y=4$ are the least integral values.

86. Proposed by B. F. FINKEL, A.M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Prove that $x^2+1457\equiv0$ (mod. 2389) is insoluble.

Solution by J. W. YOUNG, Oliver Graduate Scholar in Mathematics, Cornell University, Ithaca, N. Y., and H. S. VANDIVER, Bala, Pa.

We are to prove that -1457 is a quadratic non-residue of the odd prime 2389. Using Legendre's symbol, we must show that

$$\left(\frac{-1457}{2389}\right)=-1.$$

We have $\left(\frac{-1457}{2387}\right)=\left(\frac{-1}{2389}\right)\left(\frac{31}{2389}\right)\left(\frac{47}{2389}\right)$.

$\left(\frac{-1}{2389}\right)=+1$, since 2389 is of the form $4n+1$.

By the law of reciprocity, we have $\left(\frac{31}{2389}\right)=\left(\frac{2389}{31}\right)=\left(\frac{2}{31}\right)=+1$, since in the first place 2389 is of the form $4n+1$, and since, secondly, 31 is of the form $8n\pm1$.

$$\left(\frac{47}{2389}\right)=\left(\frac{2389}{47}\right)=\left(\frac{39}{47}\right)=-\left(\frac{47}{39}\right)=-\left(\frac{8}{39}\right)=-\left(\frac{2^2}{39}\right)\left(\frac{2}{39}\right)=-\left(\frac{2}{39}\right)=-1,$$

since 47 and 39 are both of the form $4n-1$, and 31 is of the form $2n\pm1$.

Therefore, finally, $\left(\frac{-1457}{2389}\right)=(+1)(+1)(-1)=-1$.

Also solved by *G. B. M. ZERR.*

AVERAGE AND PROBABILITY.

103. Proposed by LON C. WALKER, A. M., Assistant Professor of Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.

A circle is drawn at random both in magnitude and position, but so as to lie wholly on the surface of a given semi-circle. Show that the chance that a radius drawn at random in the semi-circle will cut the circle is

$$\frac{4}{3\pi-4}\left(1-\frac{1}{\pi}-\frac{2}{\pi}\log2\right).$$